Quasilinear sub-elliptic equations in Carnot groups

Scope of this project is to study quasilinear sub-elliptic equations in Carnot groups, with a subriemannian metric. The simplest group of this type is the Heisenberg group. This can be identified with the triple \((\mathbb{R}^{2n+1}, H, g)\) where \(H\) is subbundle of the tangent bundle and generated by:

\[
X_i = \partial_{x_i} + y_i \partial_z, \quad Y_i = \partial_{y_i} - x_i \partial_z,
\]

and \(g\) is the metric which makes these vector fields orthonormal. In a recent work Xiao Zhong [Z], have established \(C^{1,\alpha}\)-regularity (with respect to homogeneous metric) for the \(p\)-Laplacian operator in the Heisenberg group. The key idea of the proof is to establish Caccioppoli inequality which involves both \(X_i\ Y_i\) and their commutator. Once this is established, the de Giorgi method ensures the regularity result. The result has been extended to a class of subelliptic operators in [MZ] and to time dependent operators in [CCGM]. We plan to extend the result of Zhong to more general setting.

We are also interested in a more general class of equations, which come from application to the visual cortex, represented as sum of squares of vector fields. It has been proved by Bony [B] that the strong maximum principle holds also in absence of the Hormander condition, in the form of propagation of maxima. Indeed if a solution attains a maximum in an interior point, it is constant along the integral curves if the vector fields through the point. One of the scopes of the project is to understand if regularity is propagated along these vectors.


[MZ] S. Mukherjee, X. Zhong, \(C^{1,\gamma}\) -Regularity for variational problems in the Heisenberg group, preprint

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